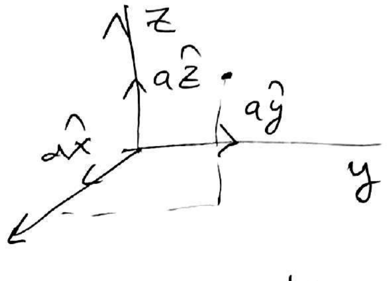
lec(2)
Waves

Coordinates System

Cartesian — cylindrical — spherical

1- Cartesian Coordinate sys (x, y, z)

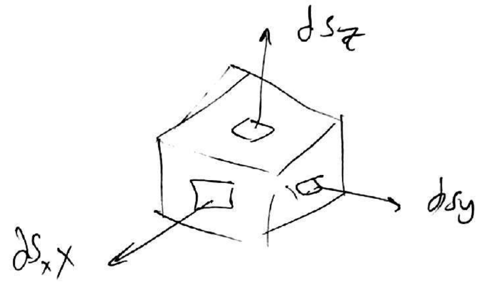


$\vec{A} = 2\hat{a_x} + 2\hat{a_y} + 2\hat{a_z}$

→ differential line (length)

$\vec{dl} = dx\,\hat{a_x} + dy\,\hat{a_y} + dz\,\hat{a_z}$

→ differential surface



$\vec{dS_x} = dy\,dz\,\hat{a_x}$

$\vec{dS_y} = dx\,dz\,\hat{a_y}$

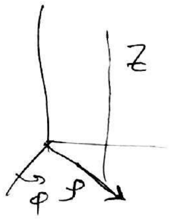
$\vec{dS_z} = dx\,dy\,\hat{a_z}$

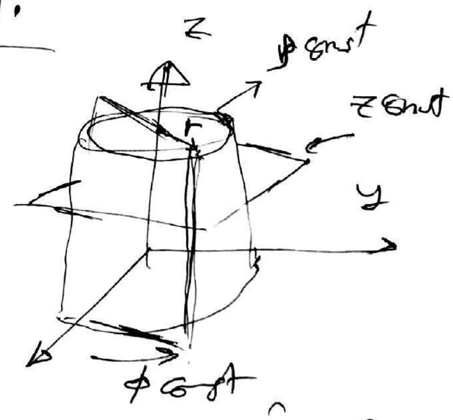
→ differential Volume $(dv = dx\,dy\,dz)$

notes $\quad -\infty \leq x, y, z \leq \infty$

2- cylindrical Coord.

(ρ, ϕ, z)





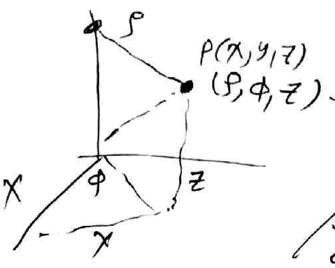
$0 \leq \phi \leq 2\pi$

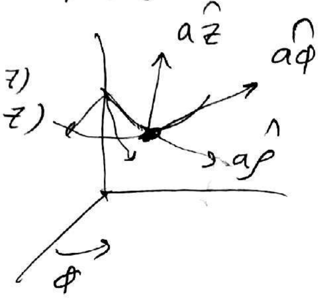
$-\infty \leq z \leq \infty$

$0 \leq \rho \leq \infty$

$\rho > 0$

$\vec{A} = 2\hat{a_r} + \frac{\pi}{4}\hat{a_\phi} + 5\hat{a_z}$





ρ ← البعد العمودي من نقطة للمحور Z

ϕ ← الزاوية مع XZ plane

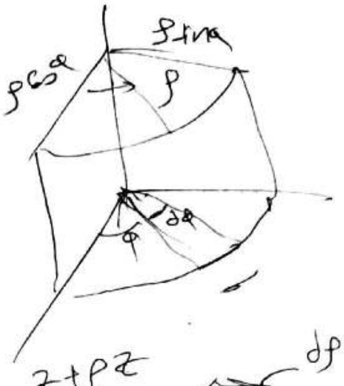
Z ← الارتفاع عن xy plane

→ differential line

$$\overline{dl} = dr\,\hat{a_r} + r\,d\phi\,\hat{a_\phi} + dz\,\hat{a_z}$$

$$|\overline{dl}| = \sqrt{(dr)^2 + (r\,d\phi)^2 + (dz)^2}$$

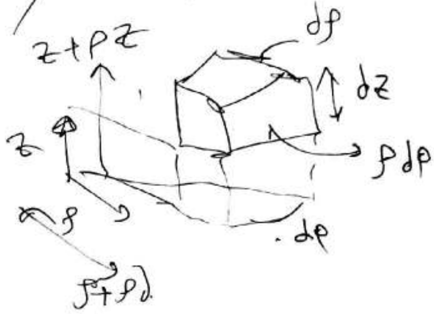
→ differential surface



$$d\overline{S_r} = \underline{r\,d\phi}\;dz \quad \hat{a_r}$$

$$d\overline{S_\phi} = dr\,dz \quad \hat{a_\phi}$$

$$d\overline{S_z} = (dr)(r\,d\phi) \quad \hat{a_z}$$

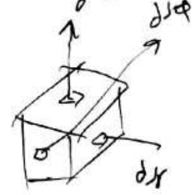


→ differential volume

$$dv = r\,dr\,d\phi\,dz$$

~~x = r cos θ~~

transformation Bet^n cartesian & Cylindrical



$x = r\cos\phi$	$r = \sqrt{x^2 + y^2}$
$y = r\sin\phi$	$\phi = \tan^{-1} y/x$
$z = z$	$z = z$

$$\begin{pmatrix} A_r \\ A_\theta \\ A_\phi \end{pmatrix} = \begin{pmatrix} & \end{pmatrix} \begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix}$$

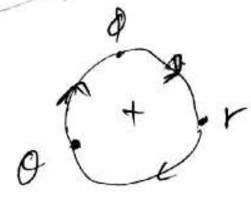
	$\overline{a_x}$	$\overline{a_y}$	$\overline{a_z}$
$\hat{a_r}$	$\cos\phi$	$\sin\phi$	0
$\hat{a_\phi}$	$-\sin\phi$	$\cos\phi$	0
$\hat{a_z}$	0	0	1

[illegible]

[illegible]

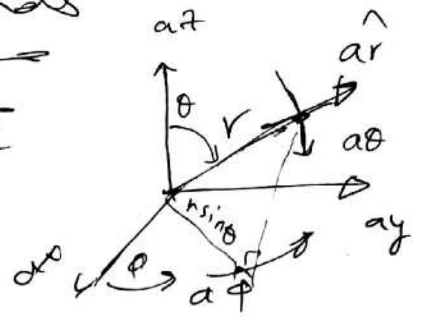
التحويل بين الاسطوانية والكارتيزية

1 2 1



3

Spherical coordinates

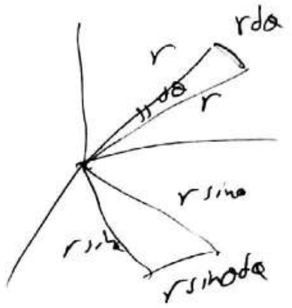


$r \geq 0$ $\quad 0 \leq \phi \leq 2\pi$ $\quad 0 \leq \theta \leq \pi$

$$\bar{A} = 2\hat{a}_r + \frac{\pi}{2}\hat{a}_\theta + \frac{\pi}{4}\hat{a}_\phi$$

→ differential line

$$\bar{dl} = dr\,\hat{a}_r + r\,d\theta\,\hat{a}_\theta + r\sin\theta\,d\phi\,\hat{a}_\phi$$

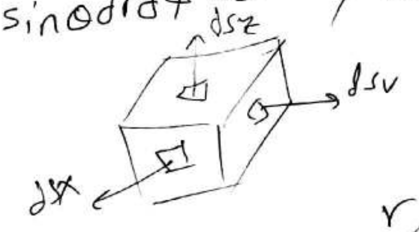


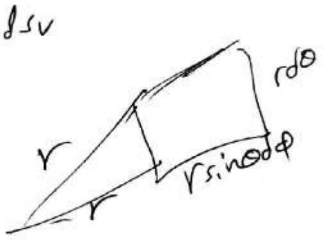
→ differential surface $\quad (r\,d\theta)(r\sin\theta\,d\phi)$

$$d\bar{S}_r = r^2\sin\theta\,d\theta\,d\phi\;\hat{a}_r$$

$$d\bar{S}_\theta = (dr)(r\sin\theta\,d\phi)\,\hat{a}_\theta = r\sin\theta\,dr\,d\phi\;\hat{a}_\theta$$

$$d\bar{S}_\phi = r\,dr\,d\theta\;\hat{a}_\phi$$





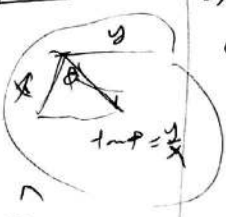
differential volume

$$dV = r^2\sin\theta\,dr\,d\theta\,d\phi$$

التحويل من Sph. لـ Cart

$$x = r\sin\theta\cos\phi$$
$$y = r\sin\theta\sin\phi$$
$$z = r\cos\theta$$

$$r = \sqrt{x^2+y^2+z^2}$$
$$\theta = \tan^{-1}\frac{\sqrt{x^2+y^2}}{z}$$
$$\phi = \tan^{-1} y/x$$



$\hat{a}_r$ اتجاهه من مركز الكرة للخارج

$\hat{a}_\theta$ المماس للكرة اتجاه زيادة θ

$\hat{a}_\phi$ المماس للكرة اتجاه زيادة ϕ

	$\hat{a}_x$	$\hat{a}_y$	$\hat{a}_z$
$\hat{a}_r$	$\sin\theta\cos\phi$	$\sin\theta\sin\phi$	$\cos\theta$
$\hat{a}_\theta$	$\cos\theta\cos\phi$	$\cos\theta\sin\phi$	$-\sin\theta$
$\hat{a}_\phi$	$-\sin\phi$	$\cos\phi$	0

تعويض بقيم الـ θ و ϕ

مستوى $\theta = 90°$ و ϕ [illegible]

$r\sin\theta$

Sheet (2) — 4 —

1) Determine the volume V of a region defined in a cylindrical coordinate system as $1m \le r \le 2m$, $0 \le \phi \le \pi/3$, $0 \le z \le 1m$ by integration. ~~check your result without performing the integration.~~

Sol

by integration

$1 \le \rho \le 2$

$0 \le \phi \le \pi/3$

$0 \le z \le 1$

$dV = \rho \, d\rho \, d\phi \, dz$

$\text{volume} = \iiint dv$

$$V = \int_1^2 \int_0^{\pi/3} \int_0^1 \rho \, dz \, d\phi \, d\rho$$

$$\therefore V = \left(\left[\frac{\rho^2}{2} \right]_1^2 \right) \left(\phi \Big|_0^{\pi/3} \right) \left(z \Big|_0^1 \right) = \pi/2 \ m^3$$

---

(2) Determine the area (S) of a surface in a spherical coordinate system as $r = 2m$ & $\pi/4 \le \theta \le \pi/3$ rad.

الكره

$0 \le \phi \le 2\pi$ مادام مجبش سيرة ϕ اعتبرها من

بـ r ثابتة فـ ds_r أخد θ, ϕ

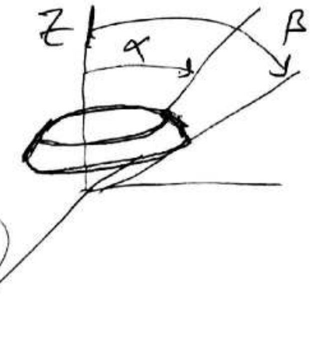
$\text{surface area} = \iint ds$

$$A = \int_0^{2\pi} \int_{\pi/4}^{\pi/3} r^2 \sin\theta \, d\theta \, d\phi = (2)^2 \left[-\cos\theta \right]_{\pi/4}^{\pi/3} \left[\phi \right]_0^{2\pi}$$

$$= 5.205 \ m^2$$

---

3) Use the spherical coordinate system to find area of strip $\alpha \le \theta \le \beta$ on spherical shell of $r = a$, show this strip by sketching. What result when $\alpha = 0$ and $\beta = \pi$



Sol

$\because r = a$ const

$\therefore ds_r = \int r^2 \sin\theta \, d\theta \, d\phi$

$0 \le \phi \le 2\pi$

$$A = \iint_{\theta\,\phi} ds$$

$$= \int_{\phi=0}^{2\pi} \int_{\theta=\alpha}^{\beta} r^2 \sin\theta \, d\theta \, d\phi \qquad r = a$$

$$= a^2 \left[-\cos\theta\right]_{\alpha}^{\beta} \left[2\pi\right] = 2\pi a^2 \left[\cos\alpha - \cos\beta\right]$$

for $\alpha = 0$, $\beta = \pi$: $A = 2\pi a^2 \left[\cos 0 - \cos\pi\right] = 4\pi a^2$ ($\to 2$)

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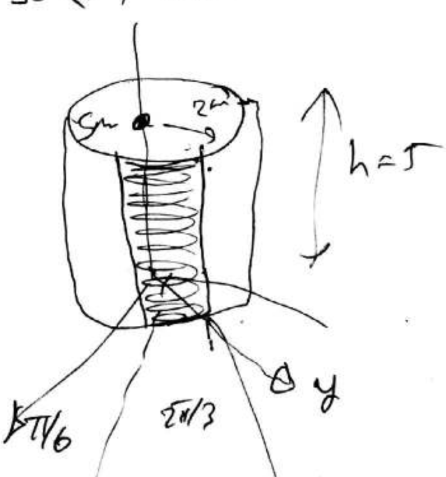
4) Use cylindrical coordinate sys. to find area of curved surface of a right cylindrical where $r = 2m$, $h = 5m$, $30° \le \phi \le 120°$



Sol

$\rho = 2$

$30° \le \phi \le 120°$ ($\pi/6 \le \phi \le 2\pi/3$)

$h = z = 5m$, $z = 0 \to 5$

area of curved surface

$\therefore \rho$ const

$$\therefore dS_\rho = \iint \rho \, d\phi \, dz$$

$$= 2 \int_0^5 \int_{\pi/6}^{2\pi/3} d\phi \, dz = 2 \times [z]_0^5 \, [\phi]_{\pi/6}^{2\pi/3} = 5\pi \, m^2$$

Curve

$dr \; r d\phi \; dz$

(5) Given point P(5m, 60°, 2m) & Q (2m, 110°, -1m)

a- Find the distance R_{PQ}

b- give a unit vector in cartesian coord. at P that directed Towards Q

~~C ... cylindrical ...~~

Sol

(a) Points in cylindrical $P(5, 60°, 2)$ $Q(2, 110°, -1)$

distance ?! R_{PQ}

Convert cylind. to cartesian

$x = \rho \cos\phi$
$y = \rho \sin\phi$
$z = z$

P

$x = 5\cos 60 = 2.5$
$y = 5\sin 60 = 4.33$
$z = 2$

$P(2.5, 4.33, 2)$

Q

$x = 2\cos 110 = -0.684$
$y = 2\sin 110 = 1.879$
$z = -1$

$Q = (-0.684, 1.879, -1)$

$$\bar{R}_{PQ} = (\bar{Q} - \bar{P}) = (-0.684 - 2.5,\ 1.879 - 4.33,\ -1-2)$$

$$= (\underset{\hat{a_x}}{-3.184},\ \underset{\hat{a_y}}{-2.45},\ \underset{\hat{a_z}}{-3})$$

$$|\bar{R}_{PQ}| = \sqrt{(-3.184)^2 + (-2.45)^2 + (-3)^2} = 5.014\ m$$

b) Unit vector Cartesian

$$a_{PQ} = \frac{\bar{R}_{PQ}}{|\bar{R}_{PQ}|} = \frac{-3.184\hat{a_x} - 2.45\hat{a_y} - 3\hat{a_z}}{5.014}$$

$$= -0.635\hat{a_x} - 0.489\hat{a_y} - 0.599\hat{a_z}$$

(6) a- Find $\hat{a_x}$ in spherical component at $P(3, -4, 5)$

يعني بدي a_x بدلالة r, θ, ϕ

b- find $\hat{a_\theta}$ in Cartesian Component at P

مع الجدول

$$\hat{a_x} = (\sin\theta\cos\phi)\,\hat{a_r} + (\cos\theta\cos\phi)\,\hat{a_\theta} - \sin\phi\,\hat{a_\phi}$$

at $p(\overset{x}{3}, -\overset{y}{4}, \overset{z}{5})$

$$\theta = \tan^{-1}\frac{\sqrt{x^2+y^2}}{z} = 45^\circ$$

$$\phi = \tan^{-1}\frac{y}{x} = -53.13^\circ$$

$$\therefore \bar{a_x} = \frac{1}{\sqrt{2}}(0.6)\,\hat{a_r} + \frac{1}{\sqrt{2}}(0.6)\,\hat{a_\theta} + 0.8\,\hat{a_\phi}$$

$$= 0.424\,\hat{a_r} + 0.424\,\hat{a_\theta} + 0.8\,a_\phi$$

(b) Cartesian

$$\hat{a_\theta} = (\cos\theta\cos\phi)\,\hat{a_x} + (\cos\theta\sin\phi)\,\hat{a_y} - \sin\theta\,a$$

$$= \frac{1}{\sqrt{2}}(0.6)\,\hat{a_x} + \frac{1}{\sqrt{2}}(-0.8)\,\hat{a_y} - \frac{1}{\sqrt{2}}\,a\hat{z}$$

$$\boxed{\hat{a_\theta} = 0.424\,\hat{a_x} - 0.565\,\hat{a_y} - 0.707\,a_z}$$

Scanned by CamScanner

Report

8

Report

7 A closed surface is defined in spherical coordinates by $3 \le r \le 5$, $0.1\pi \le \theta \le 0.3\pi$, $1.2\pi \le \phi \le 1.6\pi$

find (a) Volume enclosed

(b) total surface area

Sol

تمارين حل

$V = \iiint dv$ volume (a)

$= \iiint r^2 \sin\theta \, dr \, d\theta \, d\phi$

(b) حل السطح

$dS_r = \iint r^2 \sin\theta \, d\theta \, d\phi \quad 3 = r$

$dS_r = \iint r^2 \sin\theta \, d\theta \, d\phi \quad r = 5$

$\iint r \sin\theta \, dr \, d\phi = dS_\theta \quad \theta = 0.1\pi$

$\iint r \sin\theta \, dr \, d\phi = dS_\theta \quad \theta = 0.3\pi$

$\iint r \sin\theta \, dr \, d\theta = dS_\phi \quad \phi = 1.2\pi$

$dS_\phi \quad \phi = 1.6\pi$

$\iint r \sin\theta \, dr \, d\theta$

8 transform $\bar{A} = y\hat{a}_x + x\hat{a}_y + \frac{x^2}{\sqrt{x^2+y^2}}\hat{a}_z$ to Cylindrical

$A_\rho = \bar{A} \cdot \hat{a}_\rho = y(\underbrace{\hat{a}_x \cdot \hat{a}_\rho}_{\cos\phi}) + x(\underbrace{\hat{a}_y \cdot \hat{a}_\rho}_{\sin\phi}) + \frac{x^2}{\sqrt{x^2+y^2}}(\underbrace{\hat{a}_z \cdot \hat{a}_\rho}_{0})$

بالجدول

$y = \rho\sin\phi$

$x = \rho\cos\phi$

$\boxed{A_\rho = \rho\sin\phi\cos\phi + \rho\cos\phi\sin\phi} = 2\rho\sin\phi\cos\phi$ وهكذا

A_θ, A_ϕ نفس الحل مع الباقي